B.Sc Part II (Physics (Hons)
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Q State and deduce Maxwell's field equation in differential form or
Deduce plane e.m. waves in a conducting medium

Ans Maxwell's eq^n are gives as following form

$$\text{div } D = \nabla \cdot D = \rho$$
$$\text{div } B = \nabla \cdot B = 0$$
$$\text{curl } E = -\frac{\partial B}{\partial t}$$
$$\text{curl } H = J + \frac{\partial D}{\partial t} \quad \text{——— (1)}$$

Aussuming that medium is linear and isotropic and is characterised by permittivity ϵ, permeability μ and conductivity σ, but not any charge on any current other than ohm's law, so

$$D = \epsilon E, \; B = \mu H, \; J = \sigma E \text{ and } \rho = 0$$

Now Maxwell's eq^n in (1) becomes

$$\text{div } E = 0, \; \cancel{\text{div } H = 0} \quad \text{——— (2)}$$
$$\text{div } H = 0 \quad \text{——— (3)}$$
$$\text{curl } E = -\mu \frac{\partial H}{\partial t} \quad \text{——— (4)}$$
$$\text{curl } H = \sigma E + \epsilon \frac{\partial E}{\partial t} \quad \text{——— (5)}$$

If we take curl of eq^n (4).

$$\text{curl curl } E = -\mu \frac{\partial}{\partial t}(\text{curl } H)$$

on substituting curl H from (5), we get

$$\text{curl curl } E = -\frac{\partial}{\partial t}\left(\sigma E + \epsilon \frac{\partial E}{\partial t}\right)$$

$$\text{i.e. curl curl } E = \sigma\mu \frac{\partial E}{\partial t} - \epsilon\mu \frac{\partial^2 E}{\partial t^2} \quad \text{——— (6)}$$

Similarly, if we take the curl of (5) and substitute curl E from (4), we have

$$\text{curl curl } H = -\sigma\mu \frac{\partial H}{\partial t} - \epsilon\mu \frac{\partial^2 H}{\partial t^2} \qquad (7)$$

from vector density, we get

$$\text{curl curl } A = \text{grad div } A - \nabla^2 A$$

If we consider $\text{div } E = 0$ and $\text{div } H = 0$

$$\nabla^2 E - \sigma\mu \frac{\partial E}{\partial t} - \epsilon\mu \frac{\partial^2 E}{\partial t^2} = 0 \qquad (8)$$

$$\nabla^2 H - \sigma\mu \frac{\partial H}{\partial t} - \epsilon\mu \frac{\partial^2 H}{\partial t^2} = 0$$

These two equations represents wave equation which gives e.m field E and H in a homogenous isotropic conducting medium having conductivity σ. It is clear that these eq^n are vector equations of identical form.

In an isotropic dielectric, the time varying fields are transverse. i.e the field vectors E and H are ⊥ to the direction in which the spatial variation occurs. We know from electrostatics and magnetostatics that in the limit of zero frequency the static fields in a dielectric are longitudinal in the sense that the fields are derivable from scalar potentials and hence point in the direction of spatial variation.

No static electric fields can exist in a conducting medium in the absence of an applied current density. In case of conductors like Cu ($\sigma \simeq 10^7 \ \Omega^{-1} m^{-1}$), so that the disturbance are damped out in the extremely short time.